Network Thresholds and Multiple Equilibria in the Diffusion of Content-based Platforms.

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Abstract

This paper provides simple dynamic model of how platforms diffuse and build users community for better understanding a mechanism of diffusion process. We find that a slight difference in the initial condition will cause huge difference in the future if there is another type of network externality. We present the take-off conditions to reach positive equilibrium and we call it a critical value of the initial condition. Moreover, we build our own platform service to apply the model to a real diffusion data. As a result, the simulation based on our model can be better approximation than existing models. Finally, we discuss what is the important factor to make successful platform services.

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1 Introduction

This paper presents simple dynamic model of how online platform services diffuse and build users community. The motivation why we made the model is to understand a mechanism of diffusion process and which factor is important and to give a suggestion the way to build successful platforms based on the model. Moreover, we notice another type of network externality by focusing on the feature of platforms. As a result, our model can describe a phenomenon that platforms finally fail to launch even if it has some users in the beginning. Our main goal is to understand a diffusion process of platforms. So to explain further let us introduce a framework to analyze a diffusion process in a literature first.

1.1 Diffusion Process of Platforms

There are a lot of popular books or movies, new products, innovative ideas, and online services. However, all of them are not accepted a lot of people at the beginning, rather diffused within the population over time. Then, how are they diffused and accepted by people? To answer the question there are several fields of study and diffusion theory is one of them. This theory has been developed in a history especially in marketing, sociology and economics. One of the earliest paper is [2], that develop a simple dynamic model of adoption and diffusion process. The model developed by [2] is now widely known as the "Bass Model" and is also widely used to analyze the diffusion of new products and technologies. In the economic literature, [6] provides a social learning diffusion model and studies effect of the word-of-mouth. And [12] analyzes the contagion threshold given a network structure. Diffusion model have a lot of varieties but the common feature is focusing on the network externalities.

Network externality (also called network effect) is a concept of understanding diffusion mechanism. Network externality is the effect that one user of a good or service has on the value of that product to other people. When network effect is present, the value of a product or service is dependent on the number of others using it, that is whether I adopt or not depends on the others' actions. To understand this, consider the case of the telephone. The more people use telephones, the more useful the telephone is for each owners because if nobody use it you do not need to have a phone. So in this case, the increasing of users accelerate diffusion of the telephone.

This is an example of positive network externalities. However, there are some examples which is not fully prevailed. One of the example is Esperanto. Esperanto is a constructed international auxiliary language created by L. Zamenhof. Esperanto is designed for use of international language all over the world but Esperanto did not prevail as much as expected. It is said that Esperanto is one of the examples of negative network externality. That is, if the language is not accepted for a lot of people the number of speakers never increase.

It is widely recognized that there exists a gap between fully diffused and not diffused. The gap is known as the chasm in the fields of technology. And there is a phenomenon that once services or products reach the level of threshold then they can cross the gap. The threshold is known as the critical mass which is a concept to understand the phenomenon . When we apply this concept to above examples, we may say that population who use the telephone reached the critical mass while speaker of Esperanto did not. It is also true the other adopted process such as diffusion process of innovative technology, of the new product and popular movies. Now the word "critical mass" is used for the threshold level of the diffusion but this word originally means the smallest amount of fissile material needed for a sustained nuclear chain reaction in physics. In sociological literature, critical mass was first proposed by [15]. A lot of papers have studied this phenomenon from the perspective of sociology and economics since [15] published the monumental book. The past few years, the remarkable new models have been developed.

[20] studies the diffusion of hybrid corn in two Iowa communities using the data presented by [16] and provide a good survey that represents three kinds of diffusion model in heterogeneous populations. First one is contagion model which refers to a process in which people adopt a new product when they come in contact with others who have adopted it. Second one is social influence model which describe a situation where people have their own threshold, and once the proportion of people who have adopted they adopt. The last one is social learning model. There are a various social learning model in the sense of assumptions, but the concept is that people have their own payoff function and they adopt once the payoff for adopting is greater than the cost for it. These diffusion models is really good approximation for such a diffusion of innovations. In fact, he fits his model to the real data of [16] and obtains quite well result.

[19] also studies a critical mass from another view point. Xie et al show how the prevailing majority opinion in a population can be reversed by a small committed agents. They define a network structure and see how the actions diffuse in a population if there are small committed groups. For example, there are two political party A and B and people have their opinion which party they vote. Then, agents meet each other in a network and his or her opinion affect others' opinion one by one. They focus on how the result change if there are small committed agents, that is a committed agent has a strong opinion, for example they believe party A, and his or her opinion never change while the others' opinion can change. As a result, they show if the fraction of small community reaches 10 %, the opinion of small committed groups turns majority opinion. The remarkable result is if the fraction of small committed groups 9 % the situation does not change any more. So this is similar to a critical mass phenomenon. They refer the real situation can be seen in such as suffragette movement in the early 20th century and the rise of the American civil-rights movement that started shortly after the size of the African-American population crossed the 10 % mark.

[11] attempt to theoretically explain the late take-off phenomenon in the diffusion of telecommunication services. According to the paper, diffusion of telecommunication services is quite slow comparing to consumer durable goods.

So they compare the parameters of the diffusion process consumer durables analyzed in [2] with those of fax mart in Korea and the US. Based on their model, the shape of threshold distribution effect the take-off time. Then, they find that the late take-off phenomenon is resulted from the low heterogeneity of the threshold distribution. They also argue that the different threshold distribution cause different critical mass. These papers focus on innovative technologies (hybrid corn of [20]), opinions or ideas (Xie et al) and services (telecommunication services of [11]). But in this paper we focus on platforms especially online platform services. We think platforms have different features from others such as new products, new technologies and innovative ideas. We explain about the difference in the followings.

There are a lot of services called online platform services. Here we define "online platforms" as services which provide a place or a community where users gather and upload their contents each other. The examples are Facebook, Twitter, YouTube, Wikipedia and so on. Each of them provides different kinds of services, but common features are that their value depends on their users' activities. Consider the case of YouTube. YouTube is an online web service which provides a platform for you to create, connect and discover the world 's videos. The users can upload and enjoy their videos, but YouTube itself has no videos. So the point is if nobody upload videos, the value of YouTube will be almost zero.

This is a big difference between platforms and the others. Platforms essentially depends on contents provided by users. Let us give another examples. Nintendo and Sony provide their own video game platforms but each of the values is determined by how many softwares they have. Beta and VHS is also example of platforms. Recent example is competition in mobile platforms between iPhone and Android. Both of them provide a platform to install applications. Application is developed not only by themselves but also by third party developers. So how many applications they have is really important factor. Therefore, the essential difference of platform services is that their values will change and will determined by contents, software and applications made by their users. And if you analyze a diffusion mechanism of such a platform service, you have to consider the quality as a changeable parameter rather than a constant parameter.

However, there is a limitation when describing the diffusion of platform services. The reason why existing models are not sufficient to explain users' behaviors on the platforms fully is most of them assume the quality or value of the platform is fixed. [5] studies dynamic diffusion model considering the quality of technology might change stochastically. Their model is better in the sense of analyzing rules on thumbs but it is still not sufficient to analyze platforms. So in this paper we consider two dynamical parameter: first one is population of platforms and second one is the quality of platforms. The important difference from existing model is the quality is also endogenous variables. That is, the more people use and provide contents, the more attractive the service is. For example, the value of Wikipedia depends on how good and how many the articles are, and when considering YouTube or Twitter their attractiveness depends on how many videos or tweets and how new their contents are. So we assume the quality is determined by contents uploaded by users.

[1] develop the model which has similar features to study the dynamics open source software movements. Open source software has similar properties to platform services. The key to prevail for an open source software is not only how many users use it but also how good the software is. And the quality of a software is not constant but changes as one grows, that is, building the community is one of the most important factor. So they notice the feature and develop the good model to describe the evolution of the open source project. However, they cannot explain a critical mass. So we consider users' activities affect the others' activity through the quality. We focus on another type of network externality and find that this kind of network externality plays an important role in the diffusion of platforms.

In this paper, there are two main results. First, we show that if the conditions are not satisfied, the platform service cannot take-off and finally failed to launch even though it has some users in the beginning. And we test our model to the real situation, we developed our own music platform application for iPhone and released to iOS App Store. We gather one hundred eighty users data over five months and we find that our model gives a really good approximation.

Secondly, we suggest the three key factors based on our theoretical implication. To conclude, these three factors are how attractive the service is at the begging, how much the incentive to provide contents to a service is, and how long the contents keep attractive to the users. We provide the take-off conditions to determine successful or failure and we show these three are the main factors of take-off conditions.

1.2 Related Literature

Next we will mention about the related literature. As we mentioned above, diffusion theory is related to our paper. [15] and [2] are classical papers. In economic literature, [17] is widely known as the first paper which studies the dynamics of adoption. [5], [6], and [12] also discuss in the economic literature Our model is based on the model known as the social influence or social thresholds model. This is related to [7]. He proposed the concept of social thresholds to describe social behavior. [20] provides a diffusion model of social influence by introducing social thresholds. [18] gives an empirical study about network thresholds using three different data sets.

Platform has two different aspects in a service. One is for the users side and another aspect is for the developers or the contents providers. A lot of papers study these distinctive features of platforms. The groundbreaking work of [13] and [14] on "two-sided markets" has stimulated much theoretical work and an increasing amount of empirical work on multi-sided platform businesses that exploit indirect network effects between distinct customer groups. As we said, video game console firms, for example, realize network effects from people who buy their consoles and publishers who build games on their platforms. Another example is open source software community. There are users of open source software, on the other hand there are also developers in a community. If you manage platform business or open source software community, you should consider the aspect not only for the users but also the developers who contribute. Therefore, community design plays an important role in building sustainable platforms. On the other hand, this kind of behavior for developers is a little bit different from classical economics has long assumed because there are no monetary incentive to develop a software and to provide contents to a service. So they seems to have another kind of incentive to contribute the community. Several recent papers discuss this interesting question from various point of views.

[3] and [4] study the incentive of knowledge sharing in virtual communities. Knowledge sharing also has the features of platforms, that is there are two different segment of users. They focus on why users answer the questions and what is the factors to encourage them to share their knowledge. [8] and [9] give empirical studies about incentive mechanism of open source software community. [8] gather one hundred forty users data about developers of Linux Kernel. Linux Kernel provides a central function for Linux that is one of the most popular operating system all over the world and has market share of over 90 % as a internet server, and Linux Kernel is developed by the developers around the world. On the other hand, [9] argue the similar problem focusing on the different object. They do not focus on open source software development itself but on open source software field support system. Open sources are free software so most of these kinds of projects do not have official support for use. Even though they do not have an official support system, most of them are widely used in the world because they have user-to-user assistant. Apache, one of the most popular open source software project and used for the servers in the internet, also does not have its support system but there is very successful unofficial user-to-user assistant forum. They study why the forum does works well and why people contribute to the forum for free such as reporting bugs, answering the questions and assisting the beginners. They find that monetary incentive does not play an important role, rather developers of Apache want to help others because the others helped them before and want to get a reputation because reputation show how much they have contributed.

There are some papers in economics. [1] give an theoretical framework to describr the process of how the incentives affect the dynamics of open source software community. They consider not artistic developers but developers who have their own utility functions. [10] examines contributions to open source projects. They predict that the share of corporate contributions become more sensitive to the growth of the project than those of the an individual "hobbyists". They then test these ideas empirically using a panel data-set of activity in one hundred open source projects between 2001 and 2004. They find several patterns consistent with theoretical predictions.

Deep understanding of diffusion process of platform is beneficial for all who try to build the community where users activity can affect the others' actions each other. This is because a little difference of understanding the mechanism can cause a big difference in the end. Therefore, our model and implications give you help to grasp the situation and to make the appropriate policy. From the next section, we introduce our model setting.

2 The Model

Consider the environment of economics where there are infinitely many agents $i \in [0, 1]$ and one company which provide an online service.

The status of a online platform service at time t is depends on the number of the users and the quality. Note that in the environment, the quality of the service is not constant. Let m_t and q_t be the population who use the service and the quality of a service at time t.

The status of an agent is whether he or she uses it or not. For each agent i, suppose that there exists a minimum proportion $r_i \geq 0$ such that agent i adopts as soon as proportion of the users who are adopted reach r_i. For example, consider the case that agent i has a threshold $r_i = 0.5$ This means that agent i begins to use if more than a half of population use it. (If $r_i > 1$ represents he or she never adopts) Let $f(r)$ and $F(r)$ be the distribution function and the cumulative distribution function of thresholds in some given population.

At time t, there are m_t active users of the service, that is, proportion of agents who have already adopted is m_t. Given m_t, the proportion whose thresholds have crossed is $F(m_t)$. This follows that there are $F(m_t)$ agents who will adopt at time t. Note that $F(m_t)$ also includes the proportion who were already adopted at time t. Then the increasing of adopted agents at time t is $F(m_t) - m_t$. This is because $F(m_t)$ is the number of agents who adopts, on the other hand m_t agents have already adopted, then new users at time t will be $F(m_t) - m_t$. If $F(m_t) - m_t < 0$ the number of active users decreases at time t. Let $\lambda \in (0, 1)$ be the instantaneous rate at which these people convert. Therefore, the dynamics of mass is given by this.

$$\dot{m_t} = \lambda(F(m_t) - m_t)$$

And $m_t \in [0, 1]$ for all t.

Next, let us consider the quality of the service. We assume the quality is determined by contents provided by adopted users. And at the same time we assume the quality depress at a rate of $\beta \in (0, 1)$ over time. The intuitions of β represents if nobody provides contents the quality is decreasing at constant rate. While β can be different from service to service, we can think β as constant parameter in a service over time. So here we assume β is constant over time. Therefor, the dynamics of quality is give by this.

$$\dot{q_t} = cm_t - \beta q_t$$

And $q_t \in [0, \infty)$ for all t.

How does the quality affect diffusion processes? To see this, consider a example from a real online service "Twitter". In reality we can think that how many tweets Twitter have will affect users' decisions. In this social influence model, the distribution function represents agents' decisions whether they use or not, so we assume the current quality of a platform service will affect the threshold distribution. Moreover, if the quality of Twitter will increase, incentive to use this service will also increase because users can access good information in Twitter. For these reasons, we assume the following two assumptions.

Assumption 1. *The distribution function of thresholds is a function of* q_t *.*

First, the Assumption 1 shows that q_t will affect the distribution function so we can write the cumulative distribution function as $F(r|q_t)$, where r represents thresholds. Secondly, we assume the following property of $F(r|q_t)$.

Assumption 2. *Let $q' > q$. Quality q' has first order stochastic dominance over quality q. That is, $F(r|q') \geq F(r|q)$ for all threshold level r, with strict inequality at some r.*

The intuitions of the Assumption 2 is the followings. The distribution function represents the proportion whose thresholds have crossed. For example, given a level of quality q, $F(0|q) = 0.2$ means that 20% of population begin to use even if nobody have adopted it. Consider the case that the quality increases to q' ($> q$). Assumption 2 means that $F(0|q') \geq F(0|q)$. For example, let $F(0|q')$ be 0.3. In this case, if the quality increase to q', then additional 10% population will be adopted. So these assumptions represent the better the quality is, the more attractive the service will be and, as a result, the more people will start using it.

Therefore, we have two differential equations

$$\dot{m_t} = \lambda(F(m_t|q_t) - m_t)$$
$$\dot{q_t} = cm_t - \beta q_t$$

And given these differential equations, the diffusion path $\{m_t, q_t\}_{t=0}^{\infty}$ is determined uniquely when the initial condition (m_0, q_0) is chosen. In the following, we assume $m_0 = 0$, that is no one has adopted at the beginning.

Next, we analyze equilibrium. Equilibrium is defined as the point m_t and q_t will converge over time.

Definition 1. *The equilibrium (m^*, q^*) is defined as $m^* = \lim_{t\to\infty} m_t$ and $q^* = \lim_{t\to\infty} q_t$*

Definition 2. *Critical value of quality is defined as the level of quality $\bar{q}$ such that if $q_0 \in [0, \bar{q})$ then $\lim_{t\to\infty} m_t = 0$ and if $q_0 \in (\bar{q}, \infty)$ then $\lim_{t\to\infty} m_t = m^*$ $(m^* > 0)$*

Proposition 1. *Given c, β, $F(\cdot, q)$ and two differential equations. If $F(0|0) = 0$ is satisfied. And if there exists $\hat{q} \in [0, \infty)$ such that $(m_0, q_0) = (0, \hat{q})$ and*

$\lim_{t\to\infty}(m_t, q_t) = (m^*, q^*) \gg (0,0)$. *Then there exists a critical value of quality* $\bar{q}$.

Proof of Proposition 1. Consider $(m_0, q_0) = (0, q)$ and $(\tilde{m}_0, \tilde{q}_0) = (0, \tilde{q})$ where $\tilde{q} > q$. Let $\{(m_t, q_t)\}_{t=0}^{\infty}$ be a diffusion path when the initial condition is $(0, q)$ and $\{(\tilde{m}_t, \tilde{q}_t)\}_{t=0}^{\infty}$ be a diffusion path when the initial condition is $(0, \tilde{q})$. We will show $\lim_{t\to\infty}(\tilde{m}_t, \tilde{q}_t) \geq \lim_{t\to\infty}(m_t, q_t)$.

Consider (m_t, q_t) and $(\tilde{m}_t, \tilde{q}_t)$ where $\tilde{m}_t \geq m_t$ and $\tilde{q}_t \geq q_t$. Then we have $F(\tilde{m}_t|\tilde{q}_t) \geq F(m_t|\tilde{q}_t)$ by the definition of the cumulative distribution function and $F(m_t|\tilde{q}_t) \geq F(m_t|q_t)$ from Assumption 2. This follows $F(\tilde{m}_t|\tilde{q}_t) \geq F(m_t|q_t)$. On the other hand, we have $\dot{q}_t = cm_t - \beta q_t$ and $\dot{\tilde{q}}_t = c\tilde{m}_t - \beta\tilde{q}_t$. Let $m_{t+\delta t} = m_t + \dot{m}_t$ and $q_{t+\delta t} = q_t + \dot{q}_t$. Then, we have

$$\begin{aligned} \tilde{m}_{t+\delta t} &= \lambda F(\tilde{m}_t|\tilde{q}_t) + (1-\lambda)\tilde{m}_t \geq \lambda F(m_t|q_t) + (1-\lambda)m_t = m_{t+\delta t} \\ \tilde{q}_{t+\delta t} &= c\tilde{m}_t + (1-\beta)\tilde{q}_t \geq cm_t + (1-\beta)q_t = q_{t+\delta t} \end{aligned}$$

Therefore, $(\tilde{m}_t, \tilde{q}_t) \geq (m_t, q_t)$ holds for all t. Then we have

$$(\tilde{m}^*, \tilde{q}^*) \geq (m^*, q^*)$$

where (m^*, q^*) and $(\tilde{m}^*, \tilde{q}^*)$ are the equilibrium of $\{(m_t, q_t)\}_{t=0}^{\infty}$ and $\{(\tilde{m}_t, \tilde{q}_t)\}_{t=0}^{\infty}$ respectively.

From the assumptions we have diffusion path $\{(m_t, q_t)\}_{t=0}^{\infty}$ from the initial condition $(m_0, q_0) = (0,0)$ will converge to $(0,0)$. Moreover, there exists $(m_0, q_0) = (0, \hat{q})$ such that diffusion path $\{(m_t, q_t)\}_{t=0}^{\infty}$ will converge to (m^*, q^*).

Therefore, there exists $\bar{q} \in (0, \hat{q})$ such that the equilibrium from the initial condition $(0, q_0)$ where $q_0 \in [0, \bar{q})$ will be $(0,0)$ and the equilibrium from the initial condition $(0, q_0)$ where $q_0 \in (\bar{q}, \infty)$ will be (m^*, q^*) where $(m^*, q^*) \gg (0,0)$.

□

Example 1. *Uniform Distribution*

Consider a uniform distribution $U(a/q - 1, a/q)$ as the distribution function.

Proposition 2. *If* $c/\beta > a$ *and* $\lambda(c/\beta - a)^2 \geq ca$ *hold, then there exists a critical value of quality* $\bar{q} \in [a, c/\beta]$.

Proof. See Appendix A. □

Now we introduce a numerical specification in the case of uniform distribution function. Let $c = 0.1$, $\beta = 0.1$, $a = 0.18$ and $\lambda = 0.4$. This specification satisfies $c/\beta > a$ and $\lambda(c/\beta - a)^2 \geq ca$. From Figure.1 and Figure.2, we can see the condition of Proposition 1 is satisfied. In fact, Figure,2 shows while the initial condition $(m_0, q_0) = (0, 2)$ realize a lower equilibrium $(m^*, q^*) = (0, 0)$, diffusion path from $(m_0, q_0) = (0, 0.22)$ converges to positive equilibrium $(m^*, q^*) = (1.0, 1.0)$ in this specification. Moreover, we can find a critical value of quality $\bar{q}$ also exists within the scope of $[0.2, 0.22]$.

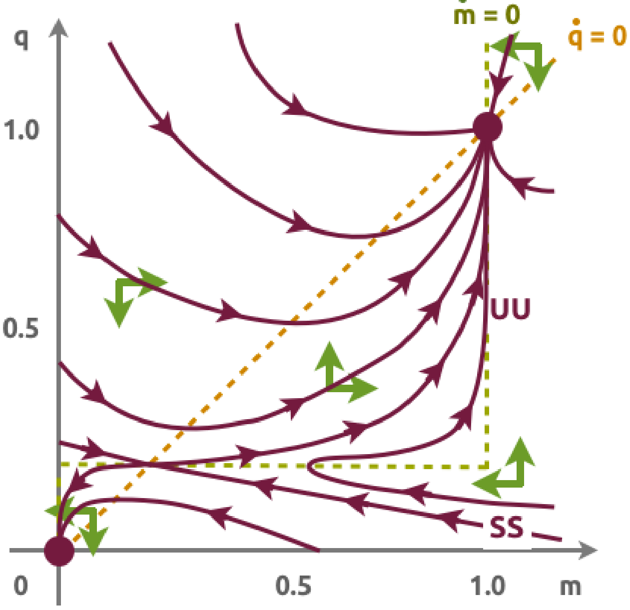

Figure 1: Phase Diagram: diagram of m_t (horizontal) and q_t (vertical) when uniform distribution $U(0.18/q - 1, 0.18/q)$, $\lambda = 0.4$, $c = 0.1$ and $\beta = 0.1$. Green line and orange line represent $\dot{m} = 0$ and $\dot{q} = 0$ respectively.

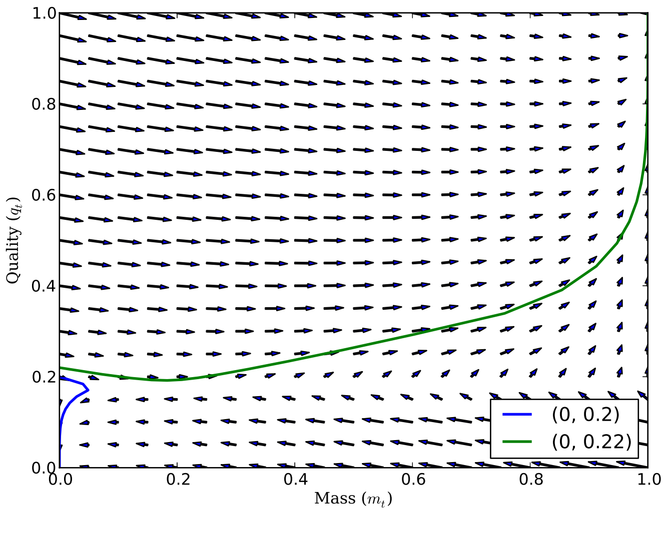

Figure 2: Phase Diagram: diagram of m_t (horizontal) and q_t (vertical) when uniform distribution $U(0.18/q - 1, 0.18/q)$, $\lambda = 0.4$, $c = 0.1$ and $\beta = 0.1$. Blue line and green line represent path of the initial condition $(m_0, q_0) = (0, 0.2)$ and path of the initial condition $(m_0, q_0) = (0, 0.22)$ respectively.

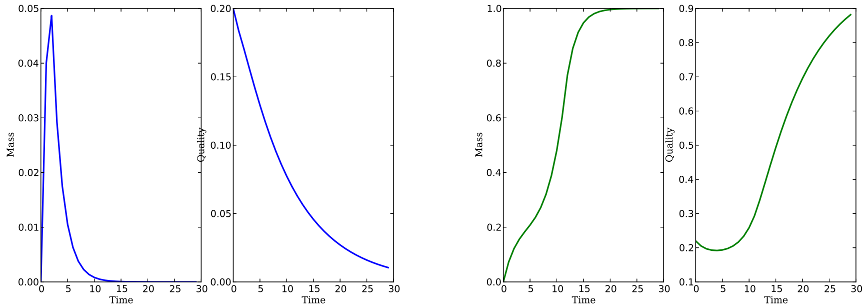

Figure 3: Adoption Curve: the dynamics of m_t and dynamics of q_t when $U(0.18/q - 1, 0.18/q)$, $\lambda = 0.4$, $c = 1.0$, $\beta = 0.1$. Left one is the diffusion process of initial condition $q_0 = 0.2$ and right one is that of $q_0 = 0.22$

Example 2. *Normal Distribution*

Consider a normal distribution $N(\mu/q, \sigma^2)$ as the distribution function. If $q > 0$ cumulative distribution function $F(\cdot|q)$ is given by this.

$$F(m|q) = \int_{-\infty}^{m} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\frac{\mu}{q})^2}{2\sigma^2}} dx$$

And we define $F(m|0) = \lim_{q \to 0} F(m|q)$ for $m \in [0, 1]$. This normal distribution function satisfies Assumption 1 and 2.[1]. Now we will apply proposition 1 to this example.

Proposition 3. *If the following condition is satisfied, then there exists $(m^*, q^*) \neq (0, 0)$ such that $\dot{m} = \lambda(F(m^*, q^*) - m^*) = 0$ and $\dot{q} = cm^* - \beta q^* = 0$ are hold.*

$$\frac{c}{\beta} \geq \min_{(\bar{m}, \bar{q}) \in A} \left[\frac{1 - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}}{\frac{\partial F(\bar{m}|\bar{q})}{\partial q}} \right]$$

where $A = \{(\bar{m}, \bar{q}) \in [0, 1] \times [0, \infty) \mid F(\bar{m}|\bar{q}) - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}\bar{m} - \frac{\partial F(\bar{m}|\bar{q})}{\partial q}\bar{q} = 0\}$.

Proof. See Appendix C. □

So if there exists such a (m^*, q^*), it is possible that there exists equilibrium of $(m_0, q_0) = (0, \hat{q})$ converges to (m^*, q^*). However, it is difficult to find such a $(m_0, q_0) = (0, \hat{q})$ analytically. Instead, we did numerical calculation and check there is also a critical value of the quality in almost all case where the above condition is satisfied.

[1] See Appendix B.

Now we introduce a numerical specification in the case of normal distribution function. Let $c = 0.1$, $\beta = 0.1$, $\mu = 0.1$, $\sigma^2 = 0.2$ and $\lambda = 0.4$. From Figure.4 and Figure.5, we can see the condition of Proposition 1 is satisfied. In fact, when you see Figure.5 you can find that there are two different paths. One is a path of the initial condition is $(m_0, q_0) = (0, 0.2)$ and the other one is that of $(m_0, q_0) = (0, 0.22)$. And you can also find the slight different at the beginning cause a large different in the equilibrium even when all of the parameter and thresholds distribution function are the same. Moreover, a critical value of quality $\bar{q}$ exists within the scope of $[0.2, 0.22]$ in this specification.

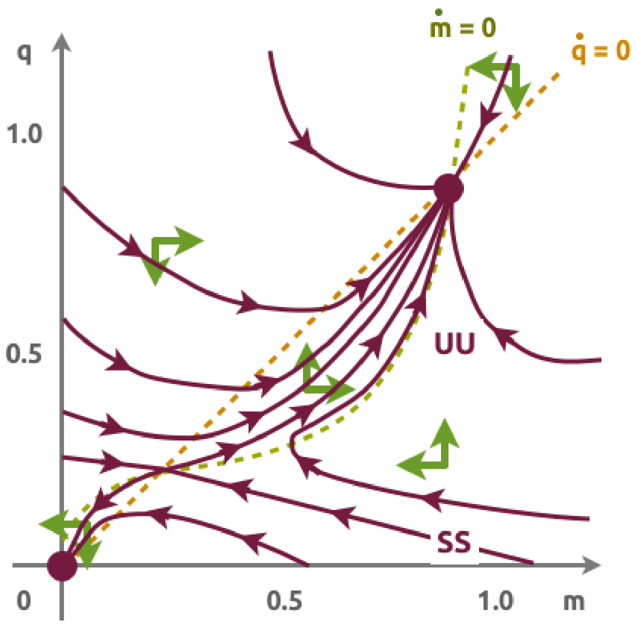

Figure 4: Phase Diagram: diagram of m_t (horizontal) and q_t (vertical) when normal distribution $N(0.1/q, 0.2)$, $\lambda = 0.4$, $c = 0.1$ and $\beta = 0.1$. Green line and orange line represent $\dot{m} = 0$ and $\dot{q} = 0$ respectively.

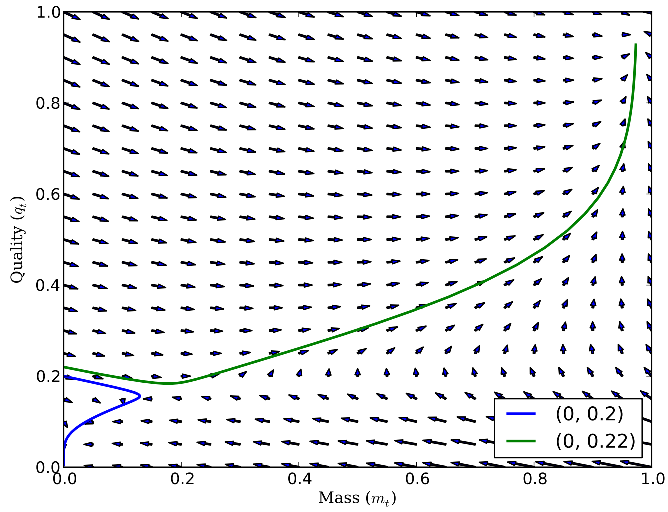

Figure 5: Phase Diagram: diagram of m_t (horizontal) and q_t (vertical) when normal distribution $N(0.1/q, 0.2)$, $\lambda = 0.4$, $c = 0.1$ and $\beta = 0.1$. Blue line and green line represent path of the initial condition $(m_0, q_0) = (0, 0.2)$ and path of the initial condition $(m_0, q_0) = (0, 0.22)$ respectively.

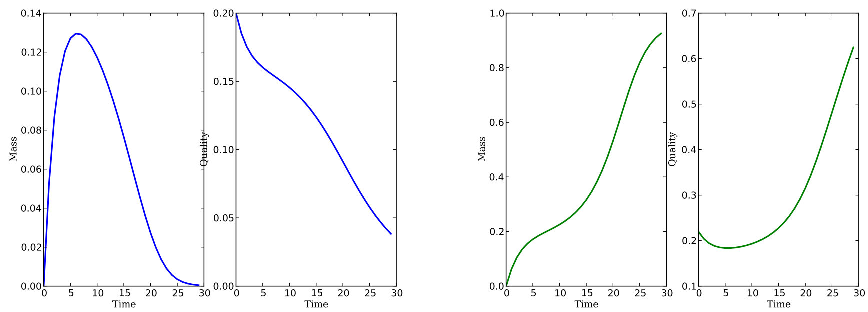

Figure 6: Adoption Curve: the dynamics of m_t and dynamics of q_t when $N(0.1/q_t, 0.2)$, $\lambda = 0.4$, $c = 1.0$, $\beta = 0.1$. Left one is the diffusion process of initial condition $q_0 = 0.2$ and right one is that of $q_0 = 0.22$

3 The Data

In this section, we will show that our model can analyze the real phenomenon better than existing models. To gather data we plan to make our own online platform service and collect diffusion data. There are several reasons for this. One reason is that it is difficult to gather the detail data about users and contents. Companies do not usually publish their data. Moreover, we can sometimes get the point data about how many users platform has now but we cannot gather the time series data. It is more difficult to access data especially at the very beginning of a service. And another reason is that it is difficult to collect data when a platform is failed because such a company disappears before publishing their data.

For these reasons, we decide to build an application about music online platform service. We will explain more detail about our application below.

3.1 The Diffusion of Music Platform Application

We design iOS application by ourselves and collect data from users' activities on our iOS application. We build our software by writing a code of Objective-C (programming language) and using XCode that is IDE (Integrated Development Environment) for iPhone and iPad application. We distribute our app via iOS App Store. App Store is designed by Apple Inc. and all of the iPhone and iPad application is distributed via here. Users can download and install to their own devices through iOS App Store. Our application can be seen in the internet (URL is `https://itunes.apple.com/jp/app/populi-social-music-app/id534963588?mt=8`) but if you try to download it, you have to use your iPhone and download from App Store. So our application is designed for only those who have iPhone or iPad.

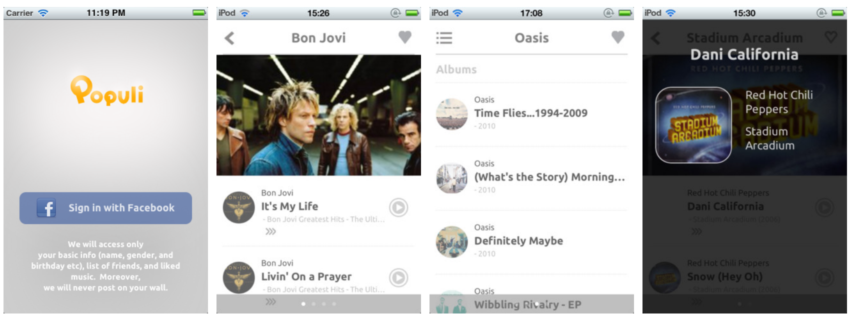

Figure 7: Screen Shot of the application

Once you install our application into your device, you have to sign in before starting using it. Sign in is taken via Facebook. We will explain for the detail about it. Facebook provides their own API (Application Programming Interface) known as Graph API to allow third party developers to access users' personal data saved in Facebook such as name, gender, location, birthday and so on. Moreover, if the users allow the developers to use their personal data, the developers can access additional information such as artists whom they likes and friends connection on Facebook. Sign in process will finish automatically once you allow our app to access your data. After signing in via Facebook you can start using it.

This application provides three functions. First one is that you can enjoy listening audition music via iTunes Music Store. You can keep listening to your favorite music sequentially in 30 seconds as a trial even if you don't have. The source of free trial music is provided by iTunes Music Store that is the largest online music store of Apple and developers can use their music source for free. Secondly, you can see and listen to music which your friends like. Our application have data of who is your friends via Facebook API so we can offer such a data. You can see friends' liked songs, albums and artists. Finally, our application provides music recommendation based on your data. Once you provide your favorite tracks, albums and artists then we calculate and pick the best one for you by data mining technology. Therefore, you can discover new music more easily and pleasantly when you use our application.

Next, we will see the relationship between the real application and the model. First, the application has the feature of a platform service because the quality partly depends on contents the users provide. We consider information about liked music as contents in the application. Users store their favorite songs, albums and artists in the application. Information will affect the value of the service because the more information the platform has, the more useful and the more enjoyable it is. In more detail, increasing of information will improve the service in the following senses. One is that you can discover new music more easily. The similarity between artists is really important factor to discover new artists. For example, if you like classic music especially Mozart you might also like Bach and Chopin rather than pop musician or rock musician. If there is no information, we cannot offer the good recommendation. So contents improve the quality in that better information leads to more accurate recommendation. Second one is that you can see more information about friends favorite. When you search good songs or artists you don't know recommendation from friends is sometimes really helpful. So accumulation about favorite music of users also improve the quality. Secondly, the quality will shrink if nobody upload new contents. In reality, contents uploaded by users never decrease but we assume the value of contents will diminish over time. We think the assumption also applies to the situation.

Summarizing above, we consider m_t as the number of active users that does not include people who download the application but do not use it. And we also regard q_t and c as the total amount of data about liked songs, albums and artists provided by users and average activities per capita respectively. We will see how the application diffuse among people in next section.

3.2 Analysis

We started from June 21st to November 21st and gather data in 5 months. We define "active at time t" as user who click like button and post data to server at time t. When user click like button on the application, then data posts to our server. We save information about user id who clicked button and time when user clicked button.

The result is shown in the below table.

Total registered user	Total like count	Average like count
185	835	4.51

Total registered users means that the number of users who downloaded and signed in the application. Total like count represents how many times users clicked like button and post data to our sever. Average like count is like count per capita.

To compare actual data of the application to our model we simulate the model in the following settings. In the simulation we assume $F(\cdot|q)$ as normal distribution $N(0.1/q, 0.2)$ and $c = 1.0$, $\beta = 0.1$. So the dynamics is given by this.

$$\dot{m_t} = 0.4 \left(\int_{-\infty}^{m_t} \frac{1}{\sqrt{2\pi}\ 0.2} e^{-\frac{(x-\frac{0.1}{q_t})^2}{0.08}} dx - m_t \right)$$

$$\dot{q_t} = 1.0m_t - 0.1q_t$$

or equivalently

$$m_{t+1} = 0.6m_t + 0.4 \int_{-\infty}^{m_t} \frac{1}{\sqrt{2\pi}\ 0.2} e^{-\frac{(x-\frac{0.1}{q_t})^2}{0.08}} dx$$

$$q_{t+1} = 1.0m_t + 0.9q_t$$

Simulation result can be seen in Figure.8 when initial condition $(m_0, q_0) = (0, 0.1)$. Left hand side of Figure.8 is dynamics of m_t and right hand side is dynamics of q_t.

On the other hand, the actual data from our application also can be seen in Figure. 9. Each graph of Figure.9 represents time series data of active user and quality.

Before seeing the graph there are several remarks. Note that active user data means that how many users is active at time t. So it is different from total number of users. Unlike the total users, the number of active users can decrease if the user stops using it. Moreover, quality represents current value of total contents. Contents increase when user upload but never decrease in reality because contents are stocked in the server. However, as we assume before quality will depress at rate of β. So the graph shows that not actual contents data but converted data using β. For example, let the amount of contents at time 0 be

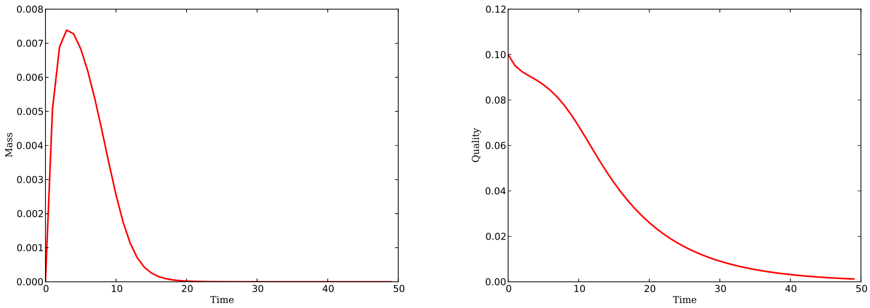

Figure 8: Simulation: dynamics of mass (left) and quality (right) when $N(0.1/q_t, 0.2)$, $\lambda = 0.4$, $c = 1.0$, $\beta = 0.1$, and $q_0 = 0.1$

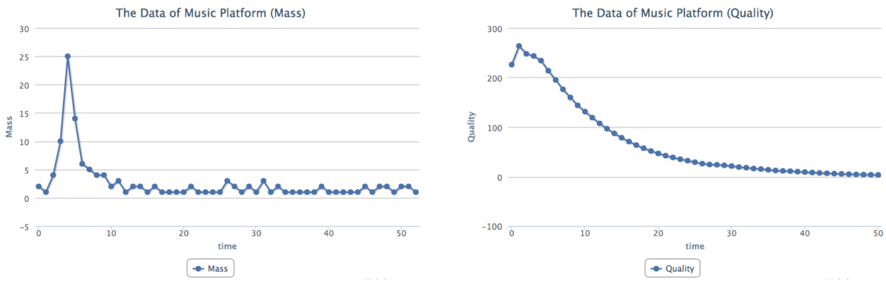

Figure 9: Actual Data: active user data (left) and amount of contents provided users assuming $\beta = 0.1$ (right) in the App from June, 21 to November, 21

10. the value of this diminishes to $10 * (1-\beta)^t$ at time t. So we calculate in the following way when we convert the real content data to quality of the model.

$$q_t \approx \sum_{\tau=0}^{t} contents_\tau * (0.9)^{t-\tau}$$

and we set 3 days as the interval of time, so $T = 50$ is about 5 months.

According to the two graphs, our model is a good approximation of the real data. Of course, it might not be convincing you because there are some parameters and specifications we assume above. If parameters such as c and β and specification of thresholds distribution change, then the simulation result might also change. However, our model succeed to explain the phenomenon the existing models cannot. For example, the social influence model of [20] is a good approximation in the successful platforms cases but his model cannot explain such a failed case. This is because there is no factor that changes threshold distribution. As a result, the mass always converges to some positive point and never decreases in his model. But it is not rare case that nobody finally use a service especially in the diffusion of platforms.

In this point of view, we improve the model and describe real phenomenon with the assumption that the quality can change and can affect thresholds distribution.

4 Conclusion

Our conclusion consists of two parts. First of all, we find that there exists threshold level of initial condition which divide the successful or the filed equilibrium in the end. We call it a critical value of the quality and Proposition 1 states about the take-off condition. According to Proposition 1, if there are different paths converge to different equilibrium there exists a critical value of the quality and under this condition platforms can never reach good equilibrium if the initial condition of quality is lower than the critical value. This is resulted from the two assumptions that the quality can change and that the quality of platform affects thresholds distribution. We made the model with these assumptions and show that the extensive model can describe the case of failed to take-off. Moreover, the result of simulation is quite good approximation of the data from real platform application. The existing models can predict the path of successful one but cannot describe the path of failed to launch because their thresholds distribution do not satisfy the condition we introduce. Namely, the difference of prediction between our model and the others is that our model can describe the situation where even though some people starts using it in the beginning, no one use it in the end. Therefore, one of the contribution of this paper is that we provide the model which can simulate the situation of both cases in one framework. It is important because there is few framework to explain the cause of the phenomenon though this failure to launch is not rare case.

Secondly, we show that there are three factors to determine the result: the quality of initial condition (q_0), the provided average contents per capita (c) and rate of diminishing the value of current contents (β). Proposition 2 and 3 state that the condition for existence of a positive equilibrium if thresholds distribution is normal or uniform. The implication about the diffusion mechanism derived from our model is the critical mass is result from both of two types of network externalities. First one is widely known as usual network externality and represent if the number of users increase the others also try to use it. This kind of network effect focus on the mass, that is how many adopted users there are and most of the papers introduce the effect into their models in some form. Another one is the network externality that implies if the quality of services increase the non-adopted users have more incentive to use it. This paper shows that the latter network externality plays an important role to describe a critical mass phenomenon and we cannot ignore this kind of effect especially when we analyze the diffusion of services which have the features of the quality might change as users participate such as online platform services. The reason why the second one has such a strong effect in the diffusion process is the combination of two effects accelerate the diffusion both positively and negatively. The positive loop occur if the quality is sufficiently high. The high quality stimulate non-adopted users to adopt, as a result, the platform can get the more users. The more users the platform has, the more contents platforms can accumulate and the higher the quality will be. So once this loop occurs, the platforms keep attractive in the future and finally reach the good equilibrium where the service survive sustainably. On the other hand, there is also negative feedback resultant from the network externalities. If the quality is not so high, the speed of reduce its quality is faster than the that of increasing of accumulation. Even though it has some quality at the beginning, once the negative loop occurs platforms cannot increase their users. In other words, the less users the platform has, the less contents it has, as a result, the less attractive the quality means nobody use it and provide contents in the end. In summary, our second contribution is that we show the a mechanism behind the diffusion process of platforms.

We believe this paper helps you to understand the diffusion mechanism of platforms better and to decide appropriate policy when you mange your own service or community.

5 Appendix

5.1 Appendix A

First we will show that if $c/\beta > a$, then there exists saddle point $(m^*, q^*) = (\frac{\beta}{c}a, a)$.

Proof. In this case, the cumulative distribution function is given by this if $q \geq a$

$$F(m|q) = \begin{cases} m - \frac{a}{q} + 1 & (0 \leq m \leq \frac{a}{q}) \\ 1 & (m \geq \frac{a}{q}) \end{cases} \tag{1}$$

and if $q \leq a$

$$F(m|q) = \begin{cases} 0 & (0 \leq m \leq \frac{a}{q} - 1) \\ m & (\frac{a}{q} - 1 \leq m \leq \frac{a}{q}) \\ 1 & (m \geq \frac{a}{q}) \end{cases} \tag{2}$$

Since $F(\cdot|q)$ is increasing function of q, then this distribution function satisfies Assumption 1 and 2. Moreover, $F(0|0) = 0$ is also satisfied.

Since $\lambda(F(m|q) - m) = -\lambda(a/q - 1)$, $\dot{m} = 0$ is given by

$$q = \begin{cases} [0, a] & (m = 0) \\ a & (0 \leq m \leq 1) \\ [a, \infty) & (m = 1) \end{cases}$$

Figure.1 depicts $\dot{m} = 0$ and $\dot{q} = 0$. Therefore, if $c/\beta > a$ holds, there exists stable state $(m^*, q^*) = (\frac{\beta}{c}a, a)$ such that $\lambda(F(m^*|q^*) - m^*) = 0$ and $cm^* - \beta q^* = 0$. □

Next we will show that if $\lambda(c/\beta - a)^2 \geq ca$ holds, then there exists a critical value of quality $\bar{q} \in (a, c/\beta)$. Sketch of the proof is the followings. We have only to show there exists a path which pass through (m, q) such that $m = \frac{\beta}{c}a$ and $q \geq a$ because such a path never converges to $(0, 0)$ from the proof of Proposition 1, so to show this we focus on the tangent of $\dot{q}/\dot{m}$. And next we show that $\partial(\frac{\dot{q}}{\dot{m}})/\partial q < 0$ and $\partial(\frac{\dot{q}}{\dot{m}})/\partial m > 0$ in $(m, q) \in [0, \frac{\beta}{c}a] \times [a, \frac{c}{\beta}]$. Therefore, we have a path of the initial condition $(m_0, q_0) = (0, \frac{c}{\beta})$ passes through $(\frac{\beta}{c}a, q)$ where $q \geq a$ if absolute value of the tangent of $\dot{q}/\dot{m}$ at $(m, q) = (0, \frac{c}{\beta})$ is smaller than $(\frac{c}{\beta} - a)/\frac{\beta}{c}a$.

Proof. Consider tangent of dynamics at (m, q) is given by this.

$$\frac{\dot{q}}{\dot{m}} = \frac{cm - \beta q}{\lambda(F(m|q) - m)}$$

And

$$\frac{\partial\left(\frac{\dot{q}}{\dot{m}}\right)}{\partial q} = \frac{1}{\lambda(F(m|q) - m)^2}\Big(- \beta(F(m|q) - m) - (cm - \beta q)\frac{\partial F(m|q)}{\partial q}\Big)$$

$$\frac{\partial\left(\frac{\dot{q}}{\dot{m}}\right)}{\partial m} = \frac{1}{\lambda(F(m|q) - m)^2}\Big(c(F(m|q) - m) - (cm - \beta q)(f(m|q) - 1)\Big)$$

From equation (1), if $mq \leq a$

$$\frac{\partial\left(\frac{\dot{q}}{\dot{m}}\right)}{\partial q} = \frac{1}{\lambda\left(1 - \frac{a}{q}\right)^2}\Big(- \beta\left(1 - \frac{a}{q}\right) + (cm - \beta q)\frac{a}{q^2}\Big)$$

$$\frac{\partial\left(\frac{\dot{q}}{\dot{m}}\right)}{\partial m} = \frac{1}{\lambda\left(1-\frac{a}{q}\right)^2} c\left(1-\frac{a}{q}\right)$$

Therefore, we have $\partial(\frac{\dot{q}}{\dot{m}})/\partial q < 0$ and $\partial(\frac{\dot{q}}{\dot{m}})/\partial m > 0$ in $(m,q) \in [0, \frac{\beta}{c}a] \times [a, \frac{c}{\beta}]$.

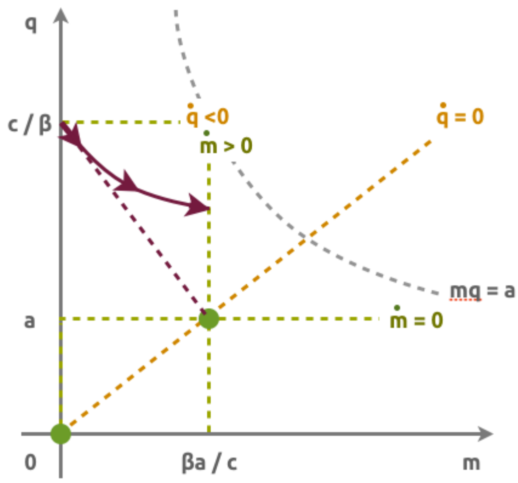

Figure 10:

Moreover, tangent of $\dot{q}/\dot{m}$ is given by this.

$$\begin{aligned}\frac{\dot{q}}{\dot{m}} &= \left.\frac{-\beta q}{\lambda\left(1-\frac{a}{q}\right)}\right|_{(m,q)=(0,\frac{c}{\beta})} \\ &= \frac{-c^2}{\lambda\beta\left(\frac{c}{\beta}-a\right)}\end{aligned}$$

Therefore, if the absolute value of the tangent of $\dot{q}/\dot{m}$ at $(0, c/\beta)$ is smaller than $\frac{c/\beta - a}{\beta a/c}$, then path started from $(0, c/\beta)$ must pass through $(\frac{\beta}{c}a, q)$ where $q \geq a$.

$$\left|\frac{-c^2}{\lambda\beta\left(\frac{c}{\beta}-a\right)}\right| \leq \frac{\frac{c}{\beta}-a}{\frac{\beta}{c}a}$$

$$\Leftrightarrow ca \leq \lambda\left(\frac{c}{\beta}-a\right)^2$$

Then, if the above condition holds $(0, c/\beta)$ never converges to $(0,0)$. This is because if $(\hat{m}_t, \hat{q}_t) \geq (m_t, q_t)$ then $\lim_{t\to\infty}(\hat{m}_t, \hat{q}_t) \geq \lim_{t\to\infty}(m_t, q_t)$ from the proof of Proposition 1. Since $(\frac{\beta}{c}a, a)$ converges to $(\frac{\beta}{c}a, a)$, $(0, c/\beta)$ converges to $(m^*, q^*) \geq (\frac{\beta}{c}a, a)$. On the other hand, $(0, a)$ also converges to $(0,0)$. Therefore, by using Proposition 1 we have there exists a critical value of quality $\bar{q}$ in $[a, c/\beta]$. □

5.2 Appendix B

Consider q and q' where $q' > q$ and let $f(m|q) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu/q)^2}{2\sigma^2}}$. We will show that for all $m \in [0,1]$, $F(m|q') \geq F(m|q)$ if $F(m|q) = \int_{-\infty}^{m} f(x|q)dx$.

Proof. If $m \in [0, \mu/q']$, then for all $x \in (-\infty, m]$ $f(x|q') > f(x|q)$. Therefore, $F(m|q') \geq F(m|q)$ is satisfied. If $m \in (\mu/q', \mu/q]$, then

$$F(m|q') = \int_{-\infty}^{\frac{\mu}{q'}} f(x|q')dx + \int_{\frac{\mu}{q'}}^{m} f(x|q')dx \geq 0.5 \geq \int_{-\infty}^{m} f(x|q)dx = F(m|q)$$

If $m \in (\mu/q, 1]$, then

$$\begin{aligned} F(m|q') &= \int_{-\infty}^{\frac{\mu}{q'}} f(x|q')dx + \int_{\frac{\mu}{q'}}^{m} f(x|q')dx \\ &= 0.5 + \int_{\frac{\mu}{q'}}^{m} f(x|q')dx \\ &= 0.5 + \int_{0}^{m-\frac{\mu}{q'}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{u^2}{2\sigma^2}} du \end{aligned}$$

$$\begin{aligned} F(m|q) &= \int_{-\infty}^{\frac{\mu}{q}} f(x|q)dx + \int_{\frac{\mu}{q}}^{m} f(x|q)dx \\ &= 0.5 + \int_{\frac{\mu}{q}}^{m} f(x|q)dx \\ &= 0.5 + \int_{0}^{m-\frac{\mu}{q}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{u^2}{2\sigma^2}} du \end{aligned}$$

This follows that $F(m|q') \geq F(m|q)$ is also satisfied. Therefore, $F(m|q) = \int_{-\infty}^{m} f(x|q)dx$ is increasing function of q. □

5.3 Appendix C

We will show that if thresholds distribution function is $N(\mu/q, \sigma^2)$ and

$$\frac{c}{\beta} \geq \min_{(\bar{m},\bar{q})\in A} \left[\frac{1 - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}}{\frac{\partial F(\bar{m}|\bar{q})}{\partial q}} \right]$$

is hold, then there exists the positive stable state (m^*, q^*).

Proof. Let $f(m|q) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu/q)^2}{2\sigma^2}}$ and $F(m|q) = \int_{-\infty}^{m} f(x|q)dx$. Since $F(m|q)$ is differentiable function of m and q the first-order approximation of $F(m|q)$ around $(\bar{m}, \bar{q})$ is

$$F(\bar{m}|\bar{q}) + \frac{\partial F(\bar{m}|\bar{q})}{\partial m}(m - \bar{m}) + \frac{\partial F(\bar{m}|\bar{q})}{\partial q}(q - \bar{q})$$

Consider $(\bar{m}, \bar{q})$ such that $\dot{m} = \lambda(F(\bar{m}|\bar{q}) - \bar{m}) = 0$ and $\dot{q} = c\bar{m} - \beta\bar{q}$. Tangent of $\dot{m} = \lambda(F(m|q) - m) = 0$ at $(\bar{m}, \bar{q})$ is given by this.

$$\lambda \left[\left(\frac{\partial F(\bar{m}|\bar{q})}{\partial m} - 1 \right) m + \frac{\partial F(\bar{m}|\bar{q})}{\partial q} q + C(\bar{m}, \bar{q}) \right] = 0$$

where $C(\bar{m}, \bar{q}) = F(\bar{m}|\bar{q}) - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}\bar{m} - \frac{\partial F(\bar{m}|\bar{q})}{\partial q}\bar{q}$. Let $A = \{(\bar{m}, \bar{q}) \in [0,1] \times [0,\infty) \mid C(\bar{m}, \bar{q}) = 0\}$. A is the set of points such that its tangent passes through the origin. Since $C(0,0) = 0$, then $A \neq \emptyset$. On the other hand, the tangent of $\dot{q} = cm - \beta q = 0$ at $(\bar{m}, \bar{q})$ is given by c/β. Therefore, if c/β is greater than the minimum slope of the tangent in A, there exists (m^*, q^*) such that $\lambda(F(m^*|q^*) - m^*) = 0$ and $cm^* - \beta q^* = 0$. □

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